2. SUMMARY OF WORK COMPLETED OR IN PROGRESS UNDER GRANT NO. N00014-92-J-1664

A. Two-layer or free surface shear flows

[1] M. Renardy, A possible explanation of "bamboo waves" in core-annular flow of two liquids, *Theor. Comp. Fluid Dyn.* 4 (1992), 95-99.

A perturbed KdV equation is derived to model interfacial waves in viscous two-layer flows. "Bamboo waves", observed in recent experiments [18], might correspond to solutions which are close to solitons.

[2] M. and Y. Renardy, Sideband instabilities in two-layer flows, *Phys. Fluids A* 5 (1993), 2738-2762.

We consider sideband instabilities following the onset of traveling interfacial waves in two-layer Couette-Poiseuille flow. The usual Ginzburg-Landau equation does not apply to this problem due to the presence of long wave modes whose decay rates tend to zero in the limit of infinite wavelength. Instead of the Ginzburg-Landau equation, we obtain a coupled set of equations for three amplitude factors. The first corresponds to an amplitude of the traveling wave, the second to a long wave modulation of the interface height, and the third is a long wave perturbation of the pressure. Criteria for sideband stability are derived. This substantially extends earlier work of Blennerhassett [19]. The coefficients in the amplitude equations are evaluated numerically for various situations, and the criteria for instability are checked and related to experiments [20]. In addition, we show the existence of (spatially) homoclinic and heteroclinic solutions. These solutions approach either a flat interface or periodic waves at $\pm\infty$.

[3] Y. Renardy, Weakly nonlinear behavior of periodic disturbances in two-layer Couette-Poiseuille flow of upper-convected Maxwell liquids, *J. Non-Newtonian Fluid Mech.*, submitted.

Studies of linear stability have revealed a major effect of fluid elasticity on the stability of fluid interfaces [21-24]. This paper proceeds beyond linear stability analysis to an investigation of the nonlinear bifurcation problem. ¿From a mathematical point of view, this is a Hopf bifurcation leading to an amplitude equation of Ginzburg-Landau type. The coefficients in this amplitude equation have been evaluated numerically for various situations.

[4] Y. Renardy, Spurt and instability in a two-layer Johnson-Segalman liquid, *Theor. Comp. Fluid Dyn.*, submitted.

A well-known phenomenon in the processing of molten plastics is the emergence of instabilities and a sudden increase in the flow rate when a critical shear rate is exceeded. One explanation which has been offered for this is that the constitutive law relating the shear stress to the shear rate is non-monotone and that a two-layer configuration arises beyond a critical shear stress [25],[26]. This paper investigates the stability of such a two-layer shearing flow to two-dimensional disturbances. The flow is found to be unstable to short enough wavelengths. Such instabilities might explain the observed surface irregularities in polymer extrudates.

[5] M. Renardy, On an eigenvalue problem arising in the study of the stability of ocean currents, *Z. angew. Math. Phys.* 45 (1994), to appear.

This paper concerns the stability of shallow-water flows in a neighborhood of the equator. Mathematically, the equations lead to a singular Sturm-Liouville eigenvalue problem. Earlier work by Hayashi and Young [27] had shown that the flow is stable if a dimensionless parameter measuring the speed of the flow (relative to a reference speed defined in terms of the north-south gradient of the Coriolis parameter and the width of the current) is larger than 3/4, while numerical calculations show instabilities for small values of this parameter. The paper gives an analytical proof of instability at all values less than 3/4.

[6] Y. Renardy and S.M. Sun, Stability of a layer of viscous magnetic fluid flow down an inclined plane, *Phys. Fluids*, submitted.

This paper concerns the linear stability of a layer of viscous magnetic fluid flowing down an inclined plane under the influence of gravity and a tangential magnetic field. While the inviscid case has been studied extensively, the viscous problem seems to have received much less attention. A recent study of weakly nonlinear long waves is given in [28]. The present paper considers linear perturbations of arbitrary wavelengths. A Squire's transformation is found, and the stability of two-dimensional disturbances is studied numerically and by asymptotics for long and short waves. The magnetic field is found to have a stabilizing effect on both surface and shear modes.

B. Viscoelastic shear flows and the relationship between stability and eigenvalues

[7] M. Renardy, On the stability of parallel shear flow of an Oldroyd B fluid, *Diff. Integral Eq.* 6 (1993), 481-489.

As is well known, a system of linear ODEs with constant coefficients is stable if all eigenvalues are in the left half plane. An abstract generalization, which is applicable to PDEs, is known for systems which generate analytic semigroups. Newtonian fluid mechanics fits into this context, but non-Newtonian fluids do not. Hence there are no general theorems available from which it can be deduced that the linear stability of non-Newtonian fluids is indeed determined by the spectrum of the linearized operator. In the paper, such a result is proved for plane Couette flow of an Oldroyd B fluid.

[8] M. Renardy, On the type of certain C_0-semigroups, *Comm. Part. Diff. Eq.* 18 (1993), 1299-1307.

This paper considers evolution problems of the form $\dot{u} = (A_0 + B)u$, where A_0 is a normal operator in Hilbert space and B is bounded. Moreover, it is assumed that the eigenvalues of A_0 cannot form clusters except perhaps in a finite region. It is proved that the type of the semigroup is determined by the spectrum of $A_0 + B$. The proof is based on a theorem of Prüß [29] and an application of the argument principle. Applications of the result lead to new and simpler proofs of exponential decay in wave equations with localized damping as well as in linear thermoelasticity.

One of the ultimate goals in investigating questions of this kind is to provide a rigorous framework for studying stability of viscoelastic flows, cf. [7] and [9]. The present result is

of interest for this because it allows to treat equations of hyperbolic type, which is clearly a prerequisite for dealing with viscoelastic flows. Unfortunately, the assumptions required on the spectrum of A_0 limit the applications to problems in one space dimension.

[9] M. Renardy, On the linear stability of hyperbolic PDEs and viscoelastic flows, *Z. angew. Math. Phys.*, to appear.

This paper has two main results. First, it is shown by a counterexample that a result along the lines of [8] cannot be generalized to more than one space dimension. Indeed, one can construct lower order perturbations of the wave equation in two dimensions which have eigenvalues in the left half plane but are nevertheless linearly unstable. From a mathematical point of view, there is a similarity between this counterexample and that of Zabczyk [30].

However, this does not rule out positive results for specific problems in more than one space dimension. The second part of the paper generalizes the result of [7]. The model is generalized to any differential constitutive equation of Jeffreys type and the flow is generalized to any parallel shear flow with strictly monotone velocity profile. It is shown that the spectrum of the linearized operator determines the linear stability of the flow.

C. Convection flows

[10] Y. Renardy, Pattern selection for the oscillatory onset in thermosolutal convection, *Phys. Fluids A* 5 (1993), 1376-1389.

A layer of fluid lies between two parallel horizontal walls and the solute concentration and temperature are higher at the bottom wall. The onset of time-periodic instability in this double diffusion problem is analyzed. Periodicity with respect to the hexagonal lattice is assumed. The no-slip condition is imposed at the top and bottom boundaries. There are eleven bifurcating solutions [31], and their stability is determined. For relatively low solutal and thermal Rayleigh numbers, the solutions are found to be unstable. For the heating of salty water, situations are presented where the standing rolls, the standing patchwork quilt, and either the standing hexagons or the standing regular triangles are stable.

[11] K. Fujimura and Y. Renardy, The interaction of oscillatory and steady modes in the two-layer Bénard problem, in preparation.

In the two-layer Bénard problem, it is possible to have simultaneous criticality of a steady and an oscillatory mode. An interesting problem of mode interaction arises when the wavelengths of these critical modes are in a ratio of 2:1 [32],[33]. It is possible to achieve this situation by an appropriate combination of a destabilizing density difference and stabilizing thermal conductivity difference. The paper exhibits such a situation and derives the amplitude equations governing the mode interaction. The bifurcation and stability of traveling waves, standing waves and steady solutions is investigated.

D. Problems with open boundaries

[12] M. Renardy, Instability of uniform flow, *Int. J. Num. Meth. Fluids*, to appear.

Numerical simulations of flow problems often involve open boundaries which arise from truncation of the domain. This makes it necessary to impose boundary conditions at these open boundaries which have no genuine physical significance. This paper explores the possibility that certain choices of boundary conditions may introduce artificial flow instabilities. The model problem studied is uniform flow transverse to a strip bounded by parallel planes.

[13] M. Renardy, Initial value problems with inflow boundaries for Maxwell fluids, *SIAM J. Math. Anal.*, submitted.

The paper considers the two-dimensional flow of an upper convected Maxwell fluid transverse to a domain bounded by parallel planes. A set of boundary conditions for the inflow and outflow boundaries is given which leads to a well-posed initial-boundary value problem. The method to construct solutions is based on an iteration which alternates between stress integration along particle trajectories and solving an equation for the velocities which is of the same type as incompressible elasticity (cf. [34],[35]). Substantial technical problems need to be overcome in order to maintain sufficient regularity during the iteration so that convergence can be established.

E. Viscoelastic flow near corners

[14] M. Renardy, The stresses of an upper convected Maxwell fluid in a Newtonian velocity field near a reentrant corner, *J. Non-Newtonian Fluid Mech.* **50** (1993), 127-134.

Numerical simulations of viscoelastic flows experience great difficulties in domains with corners larger than 180 degrees. The behavior of solutions to typical model equations, such as the upper convected Maxwell model, at such corners is not fully understood (for recent partial results see [36],[37]). This paper investigates the easier problem of determining the stresses from the constitutive relation, presuming (wrongly, of course) that the velocity field is Newtonian. The asymptotic behavior of the stresses near the corner is investigated and some numerical results are given. These results show that the integration along streamlines leads to an instability and large magnification of errors as the downstream wall is approached. This instability may be of relevance in explaining the numerical difficulties in problems with corners.

[15] M. Renardy, How to integrate the UCM stresses near a singularity (and maybe elsewhere, too), *J. Non-Newtonian Fluid Mech.*, to appear.

This paper proposes an alternative method of stress integration which avoids the feeding of errors into the unstable mode and thus the downstream blowup of computations near a reentrant corner (see [14]).

F. Breakup of viscoelastic jets

[16] M. Renardy, Some comments on the surface-tension driven breakup (or the lack of it) of viscoelastic jets, *J. Non-Newtonian Fluid Mech.* 51 (1994), 97-107.

The surface-tension driven instability of initially uniform jets shows very different characteristics in viscoelastic fluids than in Newtonian fluids. While Newtonian jets break up in finite time, viscoelastic fluids tend to form drops which remain connected by thin filaments. These filaments continue to thin out as fluid drains into the drops, but they break at a very late stage or not at all. These experimentally observed feature are also found in numerical simulations [38]. The current paper takes a more analytical approach. It analyzes a one-dimensional model which is capable of explaining these characteristics. It is shown that Newtonian jets can break in finite time, while certain viscoelastic fluid models cannot exhibit break-up in finite time.

G. Spreading of surfactants

[17] M. Renardy, On an equation describing the spreading of surfactants on thin films, *Nonlin. Anal.*, to appear.

The paper analyzes an equation which governs the spreading of a surfactant on a thin fluid film. An application which motivated earlier work on this equation [39] arises in the treatment of premature infants, where a surfactant need to be introduced into the lining of the lungs. Mathematically, the model leads to a coupled system of a parabolic equation for the concentration of the surfactant and a hyperbolic equation for the film height. The paper studies the existence of solutions to initial and initial-boundary value problems, the development of shocks, and the asymptotic stability of equilibrium solutions.